said to be necessary, and has probably under-estimated the effects on the university income of some of the proposed changes. If these effects be demonstrated, the raising of the general university grant may be expected.

The discussion of the new Bill shows that university teachers in Australia are profoundly anxious as to the present conditions, are doubtful of the possibility of maintaining the university work at an appropriate level, and are alarmed at the discontent rife among all grades of the staff. It is impossible for any one not intimately acquainted with existing conditions in Australia to judge the financial estimates, but it is obvious that if a university staff is thoroughly discontented its efficiency is bound to suffer. Teachers who are not paid a living wage must supplement their income by outside work which, except perhaps in departments of applied science, inevitably detracts from their usefulness to the university.

Those interested in the progress of the universities of the British Empire will hope that the Victorian Government, before the new Bill is passed, will allay the distracting anxieties of the university staff by making sure that the nominal increase of 30 per cent. in the general purposes grant is an actual increase of this amount, and by amending the regulations relating to the council so as to guarantee a larger representation of university teachers. The provision that one of the six members to be co-opted by the council and two of the twelve to be elected by convocation must be university teachers would give a staff representation of one in five, which is the average in the younger British universities. If by such changes the new scheme gained the confidence of the staff the reforms would restrengthen the university, which has hitherto been a glory to the State of Victoria and an important asset in the development of Australia.

Greek Mathematics.

A History of Greek Mathematics. By Sir Thomas Heath. Vol. 1, *From Thales to Euclid.* Pp. xv + 446. Vol. 2, *From Aristarchus to Diophantus.* Pp. xi + 586. (Oxford: Clarendon Press, 1921.) 2 vols., 50*s.* net.

WERE this book only for the mathematician it would be no book for me; but it is a great deal more. It is for all who care for the historical aspect of science; it is for all lovers of Greek, for mathematics is a true "Legacy of Greece," and is interwoven through and through with Greek thought and philosophy.

A couple of accidents in boyhood (if I may be allowed the reminiscence) made this subject curiously attractive to me. George Johnston Allman, pioneer in this country of the renewed study of Greek geometry, belonged to the little band of scholarly professors who taught in those days in Galway College; he had more of mathematics and more of fine historic sense than of Greek letters, and here his colleagues—"Tom" Maguire the Platonist, Davies the editor of the *Eumenides*, and my father, who was his kinsman—combined to help him. In our quiet Galway life we heard for months together little else than of Allman's book and the long discussions which went to the making of it.

About the same time I had for schoolmaster in Edinburgh (and long after for a close friend) Dr. John Sturgeon Mackay, as deeply versed in Greek mathematics as Allman himself. In Edinburgh he was scarcely known save to his schoolboys, and lived a life as neglected as that still greater Hellenist Veitch (of the "Greek Verbs") had done; but he spent his vacations reading Greek MSS. in the great libraries, and in Paris he was intimate with Paul Tannery and the scholars of the day. Mackay's life-task was to edit Pappus, the one great Alexandrine omitted by the Oxford editors of the eighteenth century. After twenty years' work the thing was done, and lay in his desk with every diagram exquisitely drawn and every page and footnote written and rewritten. He went one day into Williams and Norgate's bookshop, then in Edinburgh, and the manager said: "I have something that will interest you to-day, Dr. Mackay"; and he handed him the first volume, fresh from Leipzig, of Hultsch's edition of Pappus. The two men must (as Mackay told me afterwards) have been following one another unawares from library to library, collating the same MSS., noting the same minute textual details, and Mackay's intimacy with the French had helped doubtless, in those long-ago post-war years, to keep him unacquainted with the German and his work. Anyway, there was an end of the matter. Some might have hurried into print, trusting to little points of their own, claiming something of the reward—but not Mackay. The work was done and well done. Pappus was edited. It was a scholar's tragedy—and nothing more!

Sir Thomas Heath has produced one important work after another, while living all the while a life of strenuous official toil and responsibility. His Diophantus, his Apollonius, and (greatest perhaps of all) his Euclid are part of the solid foundation of this "History of Greek Mathematics." He has had more to build on by a long way than Allman and Gow had fifty years ago; new texts have been edited, and men like Heiberg, Tannery, Zeuthen, Aldo Mieli, and (last but not least) Gino Loria have dealt with the matter in part and whole; but the essential difficulty remains of piecing together the broken stones.

In our first glimpse of ancient mathematics two great figures, Pythagoras and Plato, stand out above all the rest: not as the greatest mathematicians, but as the greatest landmarks in history and tradition, much like Descartes and Leibniz in another age. From Plato on, the story runs fairly smooth; we leave tradition behind and enter upon history; mathematics becomes a science of its own and slowly disentangles itself from philosophy; the "polymath" is giving place to the specialist. With Euclid we enter on the "Golden Age" of geometry cradled in Plato's Academy, and though some of the minor men (like Nicomedes and Diocles) remain shadowy figures, we have a solid inheritance in Euclid and Apollonius and in Archimedes—"ordine quidem tertius" (as Torelli said), "dignitate facile princeps." On the physical side the astronomy of Eudoxus, the Optics of Euclid, the astronomy again of Aristarchus—the "Copernicus of antiquity"—may be taken as "nails in a sure place"; and when we come to the "Silver Age" of Diophantus, Pappus, Ptolemy and Theon, we have a wealth of historical material.

But up to Plato's time it is a very different story, for the simple reason that very few of the earlier philosophers and mathematicians (say down to Archytas) ever *wrote* at all. Their teaching was private or even secret, and only oral tradition carried it on. Two things men will continue to discuss but never know for sure—the debt the first Greek mathematicians owed to Egypt and the East, and the real, actual attainments in mathematics of men like Thales, Pythagoras, Archytas, Democritus and Plato. Pythagoras is the strangest case of all, for no wise man's name since Solomon's has so fascinated the world. Tradition, and tradition alone, tells us that he invented the forty-seventh proposition, and tells it no more confidently than the Dancing Dervishes of Constantinople tell you they are the descendants of his holy Brotherhood!

Where there is nothing but tradition to go upon, the historian is lost. The imaginative man begins putting two and two together, arguing what *must* have been known in order to know this or that, and what *must* have followed as soon as this or that was understood; and so the story grows. The construction of the Pentagon "must" have implied a knowledge of the Golden Section, and this and the "Theorem" itself must have involved the concept of the Irrational; the triangle of the Pentalpha and its gnomons must have led on to the logarithmic spiral—even to the Limaçon of Pascal! In some such way Naber, for instance, discourses on what Pythagoras "must" have known and what he may have known; it is pleasant, even suggestive, reading, but it is a long, long way from history. At the other end stand the sceptical critics, to be taken more seriously—like Eva Sachs, whose book, by the way, on "Die fünf platonischen Körper," Heath does not seem to quote. She, for example, holds that up to Philolaus we know nothing at all; that he and later Pythagoreans were chiefly bent on ascribing each new thing to the old master of their school; that there is no proof that Pythagoras knew anything of irrationals, little that he was acquainted with the regular solids, and none at all that he associated them, *more Platonico*, with a theory of the Elements. Between such opinions Sir Thomas Heath steers a careful middle course, and what he has to tell he attributes to "the Pythagoreans" rather than to Pythagoras.

Again, as to alien sources behind early Greek mathematics, one school will tell you that Greece had nothing or next to nothing to gain from the land-surveyors or "rope-stretchers" of Egypt, nor from Mesopotamian astronomers and calendar-makers. Others eagerly pick up little stray hints of a community or descent of ancient learning. They remind us that it was in Ionia that Greek philosophy arose and the special sciences, even medicine, began—in a "mélange de races d'émigrés, d'origine diverse," as Heiberg called the Ionians the other day; that Ionia was in close and constant touch with Lydia; and that Ionian science appeared after a clash of empires and fall of cities, as a later Renaissance followed the fall of Constantinople. Or they catch hold of straws which point, or seem to point, to Far Eastern intercourse, such (for instance) as that mode of reckoning by myriads and myriads of myriads which the Japanese are said to have used about the time (say) of Thales, which exists in China to this day, and seems identical with Archimedes' famous numeration of the *Arenarius*—where he began by supposing a myriad grains of sand in the space of a poppy-seed (or rather surely a poppy-*head*), and went on to myriad-myriads of units, and of orders, and of periods. Heath discusses the broad question briefly and fairly, and is content in the end to agree (as we all must) with Plato, that whatsoever the Greeks had borrowed, were it much or little, they it was who improved on it and carried it towards perfection. The Greeks at least knew well what not to borrow, and striking above all else is their choice of themes. It is they who best exemplify what Sir John Herschel laid down (perhaps rightly) as a general proposition—that men delight to escape from the trammels of earth: that not practical problems but "the abstractions of geometry, the properties of numbers, the movements of the celestial spheres, whatsoever is abstruse, remote and extramundane, become the first objects of infant science." The useful applications, mechanical inventions, follow later on; and David Hume put it neatly when he remarked that "we cannot reasonably expect a piece of woollen cloth to be brought to perfection in a nation that is ignorant of astronomy."

Two things attract the general student, I think, more than any others connected with Greek mathematics—the Pythagorean arithmetic and the matter of the Platonic bodies. The former begins with the notion, characteristically Greek rather than peculiarly Pythagorean or Platonic, of arithmetic as something apart from mere calculation or the doing of sums—as, in short, a "theory of numbers"; and not the least curious thing about it is that that arithmetic was studied (or so it is said) for generations before the Greeks had signs, even alphabetic ones, for the numerals. We may go to Sir Thomas Heath for a clear and full account of all the curious sorts of numbers, figurate and other, odd and even, square, triangular, and pyramidal, friendly numbers, perfect numbers, and so forth, which fertile imagination along with true mathematical insight was able to discover. It is not without a deep meaning, I believe, that we find on the very threshold of Mathematics this instinct for the symmetry of numbers, this sense of the intrinsic beauty, the comparative perfection, of one number or another. It is the way the calculating boy begins; we had it exemplified in the highest degree, only the other day, in Ramanujan's extraordinary but too short-lived talent. The theme, in Greek hands, leads on and on by many roads. By way of the "means," it is at the root of the theory of music itself, of the "acousmatic" side of the Pythagorean philosophy; by the theory of "gnomons" it is close-linked with the "theorem" of Pythagoras; it carries us, though more in contrast than identity, to the Euclidean treatment of arithmetic; and at last it brings us straight to the Neo-pythagoreans, to the "Theologumena," and to later writers down even to Kircher, who dealt more and more extravagantly (much after the fashion of the Cabbala) "de abditis numerorum mysteriis"—with the physical, the "ethical" and the "theological" properties of numbers.

The other matter, that of the Platonic bodies, is a long story. We know that Plato did not discover them but we may still be curious to know whether Pythagoras did; and here we must distinguish the mathematical side of the question from the physico-philosophical one—from the deeper meaning which Plato and others found in these five symmetries. It has been amply shown, I think, that their association with the elements was not due to Pythagoras, and it is not likely that Pythagoras knew very much even about their construction and properties. To suppose him to have understood these is to credit him with too much; the main teaching of Euclid would have been already his: "Le cadre était déjà celui," as Tannery says, "que remplissait les Eléments [d'Euclide]." But who were the great mathematicians who investigated them? Theætetus was probably the outstanding man—he who "*described*," that is *constructed*, the five solids, according to Suidas—though Heath hesitates (curiously) over the meaning of ἔγραψε. Heath quotes also the Euclid-scholion that Theaetetus added the octahedron and icosahedron to the other three, but I do not think he mentions that this has (or so it might seem) a textual flaw; for surely the octahedron was as old as the Pyramids, while the *dodecahedron* would be one of the last, probably the last of all, to be constructed and explained. Plato, then, may have taken his mathematics in this matter from Theaetetus, partly (some would say) from Democritus, and something more straight from Leucippus. So with their help was built up Plato's theory, fanciful no doubt but very beautiful, of these five figures, all inscribable in spheres, not really solids but hollow shells with filmy surfaces, made out of tiny triangles—as the gold-beater begins with little three-sided patins of gold—the figures being, as it were, molecules with the facets for atoms, and the whole forming a sort of foamy, cellular structure, like a froth of soap-bubbles, out of which to build the material of an harmonious world. Indeed, one wonders whether Plato had not in his mind's eye the homely but exquisite configuration of a froth of soap-suds!

The theme is kindred to our last. For just as an arithmetic grew up regardless of practical reckoning and dealing only with the symmetrical properties of numbers, so did a geometry arise which thought nothing of practical mensuration, only of the abstract properties, the essential symmetries, of planes and solids. This geometry, which studied the triangle, the square, the pentagon, etc., then the "Platonic" and "Archimedean" bodies, the regular and semi-regular solids, the perfect, the less perfect and the imperfect geometrical forms, was own sister to that arithmetic which investigated the triangles, squares, polygons, pyramids and cubes, the "perfections and imperfections," which lie hidden among the mysterious properties of numbers. And all the while these theoretical studies of *configuration* were being applied along somewhat narrow but very important lines to music, to optics and to astronomy, as we should say to problems of sound, light and periodic motion—in short, to the three great recognised groups of harmonious natural phenomena. This, then, in a word, was the concept of Greek mathematics as it occupied the wit of man, the intellect of philosophers, for just a thousand years.

The Archimedean bodies, by the way, Heath deals with pretty fully, but he might perhaps have told us that one of these (Kepler's "truncated octahedron"), said to have been known to Plato, is no other than that tetrakaidekahedron which Lord Kelvin showed to be (with a slight modification) the typical "cell" of a homogeneous froth. He might even perhaps have told us a little of how Kepler (true disciple of Pythagoras and of Plato) used both Platonic and Archimedean bodies in that treasure-house of elegant geometry, the "Harmonice Mundi"; and how he showed that not only the five Platonic bodies (as Euclid knew) but also Archimedes' thirteen were all there are, the complete series of their respective families.

There is a vast deal of information in Heath's book, clearly set forth and orderly arranged; we have nothing to compare with it in English, and Gino Loria's "Scienze esatte" is its only serious rival abroad. I am inclined to think that Loria paints history with a broader brush, while Heath excels in his account of individual mathematicians; but I cannot help thinking that Heath, who has attained such complete and acknowledged success in his editions of Euclid and the rest, must have found that in this history he had struck a harder task than any he had tried before. We may know more of the history of mathematics than of any other science, but the lacunæ are immense, and tradition is poor material for the historian. Moreover the historical aspect is somewhat uncongenial to the mathematician, if only because (as Eva Sachs says) history deals with *das Werden*, and mathematics with *das Sein*!

When Sir Thomas Heath deals with Euclid, Apollonius, Archimedes, Diophantus, Hero or Pappus, he gives us in a few pages all we could expect by way of epitome of the trend, the method and the results of their labours. But his book pursues its steady, instructive course with little digression, allusion or anecdote, and with curiously little bibliographical information such as he puts abundantly into his other books. Surely one of the objects of a text-book is to guide the student to what it does not and cannot contain! Some of us, I think, would have liked a little more digression or even gossip. When Sir Thomas has told us that the Pentalpha was the Pythagorean symbol of Health he is well-nigh done; but Chasles gives us a dozen pages of learned gossip upon it, traces it through Boetius and Thomas Bradwardine and the Margarita Philosophica and Father Kircher to Kepler himself, and ends with Poinssot's "Mémoire sur les Polygones"! The Shoemaker's Knife is a beautiful and simple construction in easy geometry, of great antiquity—an ancient proposition, Pappus calls it—and Heath tells us doubtless all that is essential for us to know; but a short footnote might have told us how Jacob Steiner investigated and elaborated it, or how J. S. Mackay epitomised its many properties, or how Sir Thomas Muir added a pretty corollary.

Again (as a random instance) Heath discusses at length the simple but important rule of Thymaridas (simplicity itself in our notation) for solving certain simultaneous equations, where the sum of $x_1+x_2+ \ . \ . \ . \ x_n$ is known, and also the successive sums of x_1+x_2, x_1+x_3, etc.; but of Thymaridas he only tells us that he was "an ancient Pythagorean, probably not later than Plato's time." If we be limited to a phrase I do not know that we could say a safer thing; but why should we not have some little sign-post, even a footnote, to Tannery's discussion (in "L'Arithmétique pythagorienne") on who Thymaridas was and when he lived, or to the many discussions by Cantor, Martin, Nesselmann, and even Fabricius; for "il y a un assez grand intérêt historique à déterminer l'âge où vivait Thymaridas." Heath tells us that this rule of his was called by the special name of ἐπάνθημα, and he translates it "the 'flower' or 'bloom' of Thymaridas." He qualifies this by a parenthetic remark that the name was not, after all, confined to this particular proposition, but what it really means he does not explain; Tannery, I think, has shown fairly clearly that it was a name ("pour ainsi dire") "pour les matières non exigées du programme de l'arithmétique pour les étudiants en philosophie."

It was again Thymaridas who defined (as Heath tells us) "a unit as 'limiting quantity,'" περαίνουσα ποσότης. It was a very important definition, but was it not a definition of "unity" rather than of "a unit," and is a limiting *quantity* a fair and full translation of ποσότης? Turn towards the other end of the volume, to ground that is peculiarly Heath's own, and see Euclid's famous definition (V. 3) of *ratio*, which Heath renders "a sort of relation in respect of *size* (πηλικότης) between two magnitudes of the same kind." I cannot help thinking that, between the two, we lose the fine and even crucial distinction between ποσότης and πηλικότης. The one mathematician was talking of a relation between numbers, the other of a ratio between any two magnitudes; I think they both picked their words accordingly, and I should like at least to give them the benefit of the doubt. Of Euclid's definition Heath tells us that "it was probably inserted for completeness' sake, and in order merely to aid the conception of a ratio." All the same, I should rather like to hear what Barrow had to say of its metaphysical character; or what an older school meant when they translated πηλικότης by *quantuplicity*; or even to be referred to that very curious imaginary discussion of this very point, by Euclid, Eutocius, Theon and the rest, in the pages of Meibom's "De Proportionibus." The simple fact is that Sir Thomas Heath has given us so much, and it is all so good, that he makes us ask for more.

D'ARCY W. THOMPSON.

Entomology and Malaria.

The Prevention of Malaria in the Federated Malay States: A Record of Twenty Years' Progress. By Dr. Malcolm Watson, with contributions by P. S. Hunter and A. R. Wellington. Second edition, revised and enlarged. Pp. xxviii + 381. (London: John Murray, 1921.) 36*s.* net.

DR. WATSON'S book shows clearly the wide range of scientific knowledge which is required by those who work in the tropics either as physicians or as sanitarians. It is unnecessary nowadays to insist upon the importance of the control of malaria in the development of those vast areas from which is derived so much of the food supplies and raw materials of manufacture of all civilised countries, but only those who have had practical experience of the methods used to deal with the disease can appreciate how many and how varied these must be.

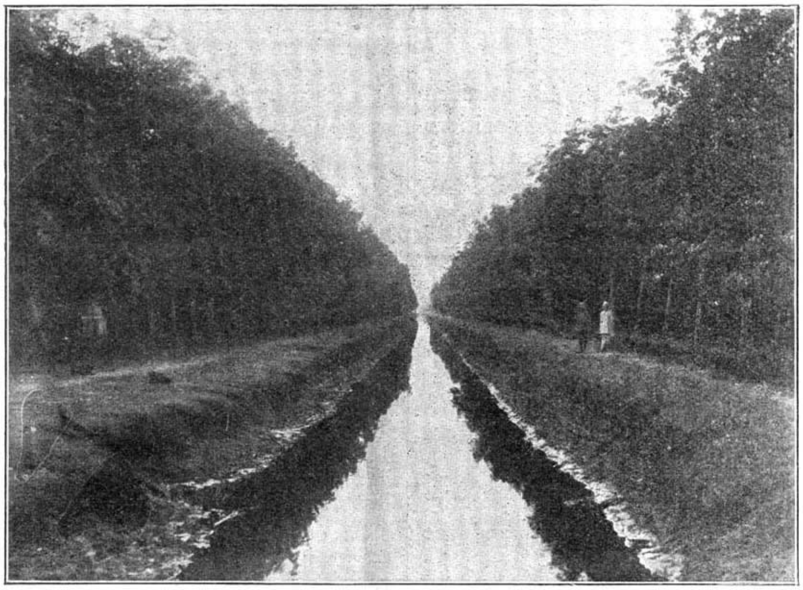

FIG. 1.—Kapar Drainage Scheme. One of the main drains, with rubber-trees on both sides. From "The Prevention of Malaria in the Federated Malay States."

Until about the middle of the nineteenth century medical men were almost always naturalists as well, and it is regrettable that the old traditional association of medicine and natural science has been so largely broken off. In modern times the older teaching of natural history has been replaced by an inadequate course of so-called biology which may, indeed, enable the student to recognise the appendages of the cockroach, or a section of the rhizome of a fern, but scarcely qualifies him to name correctly the commonest insect or plant when he sees it in the field. The need for a more practical knowledge of the forms and bionomics of animals, and especially of insects, is brought home to one again and again during the perusal of Dr. Watson's very interesting and readable book.

It was indeed no light task which confronted the author when in 1901 he began his service in the Malay States. The new knowledge of the mode of transmission of malaria by certain mosquitoes had to find a practical application in a country where physical conditions are very diversified and little or nothing was known about the insect carriers. Large commercial interests, too, were involved, and no doubt there were many interested persons ready to criticise adversely any failure, and reluctant to spend money on what they regarded as the doctor's theories. Here were the most favourable conditions imaginable for mosquitoes—an equable, warm, and moist climate; a large rainfall almost equally distributed throughout the year; abundance of pools, swamps, and hill-streams. As to the prevalence of mosquitoes, Dr. Watson relates that, in a small patch of jungle in the town of Klang, *Anopheles umbrosus*, a natural carrier of malaria, was present in such large numbers that three persons caught about two hundred in a quarter of an hour "and simply could not stand the biting any longer." At the same time, three other species of mosquitoes were present "in considerably greater numbers than the anopheline, so it can be imagined a quarter of an hour in that jungle was unpleasant."